

The Negative-Conductance Slot Amplifier*

M. E. PEDINOFF†

Summary—It has been suggested that the incorporation of active solid-state devices into the elements of an antenna may lead to the simplification of the over-all microwave system and at the same time to a reduction in size, power and weight requirements. This paper will discuss several approaches to a study of the microwave properties of a slotted antenna element shunted by a tunnel diode biased into its negative-conductance region. The first approach involves calculation of the lumped parameter equivalent circuit of the slot amplifier system at resonance and can be extended to determination of the gain bandwidth and noise performance of the device, whereas the second approach is concerned with the admittance of the slot and the diode as a function of frequency over a wide frequency range. The latter method of analysis successfully predicts the conditions of oscillation and amplification at fundamental as well as higher frequency resonances and leads to a method for stabilization of the system. Preliminary experimental results indicate a transmission gain of 21.5 db at 2.7 kMc with a Hughes Number PC-3 GaAs tunnel diode in the slot amplifier.

I. INTRODUCTION

DURING the past few years, amplification techniques and devices available to the systems designer have been improved steadily to the point where the concept of low noise amplification within the elements of an antenna could be translated into a workable system. Several systems^{1,2} reported in the literature have demonstrated the feasibility of obtaining parametric amplification and conversion gain by the use of solid-state diodes integrated into the antenna configuration; thus amplification occurs in the region of reception, and transmission losses which degrade system performance are eliminated. These devices have been used with log periodic arrays and cylindrical wire antennas. This paper describes the application of an amplification technique which uses the negative conductance of a tunnel diode in a linear magnetic radiator, namely the waveguide endplate slot antenna.^{3,4} An interesting property of the endplate slot is that when it is operated at resonance, it can be used with slight adjustment to couple all of the energy in the incident wave out into space, or it can provide unity

* Received by the PGM TT, June 8, 1961; revised manuscript received, July 31, 1961. The research reported in this paper has been sponsored by the Electronic Res. Directorate of the AF Cambridge Res. Ctr., Air Res. and Dev. Command, Bedford, Mass., under Contract No. AF 19(604)-8386.

† Hughes Aircraft Co., Malibu, Calif.

¹ A. D. Frost, "Parametric amplifier antenna," Proc. IRE, vol. 48, pp. 1163-1164; June, 1960.

² C. H. Boehnker, J. R. Copeland, and W. J. Robertson, "Antennaversers and Antennafiers—Unified Antenna and Receiver Circuitry Design," *Abstracts of 10th Annual Symp. on USAF Antenna Research and Development Program*, University of Illinois, Urbana, sponsored by Wright Air Dev. Div., Wright AFB, Dayton, Ohio, October 3-7, 1960.

³ A. F. Stevenson, "Theory of slots in rectangular waveguides," *J. Appl. Phys.*, vol. 19, pp. 25-38; January, 1948.

⁴ W. H. Watson, "The Physical Principles of Waveguide Transmission and Antenna Systems, Clarendon Press, Oxford, Eng., p. 89; 1947.

coupling to another waveguide. Thus, by placing a negative conductance element within the slot, one can obtain transmission gain through the slot for the case of electromagnetic radiation coupled to the waveguide from space or from another waveguide. Due to the bilateral nature of the slot-diode system, one obtains gain for waves incident on the slot from either direction; but in addition, the presence of the diode within the otherwise matched slot produces a mismatch and an attendant reflection coefficient greater than unity in the direction of incidence. In many applications such as a waveguide line amplifier, the large reflected component of the wave can again reflect from regions of impedance mismatch and give rise to feedback oscillations unless proper isolation is used; but for the antenna receiving application, the wave reflected from the slot is radiated into space, thus eliminating one source of instability. This ultimately results in a reduction of the available gain by approximately 3 db, but saves considerable weight and space by the elimination of the need for a circulator and at least one isolator.

II. THEORY

Waveguide Endplate Slot

Tunnel Diode in an Endplate Slot: Consider the antenna system formed by placing a tunnel diode at the center of a horizontal centered slot cut in an infinite endplate which terminates a waveguide supporting the TE_{10} mode, as shown in Fig. 1. Sevenson³ has shown that transverse broadwall slots present a series impedance to the dominant mode in the guide and can be represented by an equivalent circuit consisting of a lumped inductance, a lumped capacitance and a radiation resistance all in parallel at the frequency of resonance of the slot. Watson⁴ has shown that if the waveguide is terminated by a thin perfect conductor and the series slot is moved from the broad wall to the center of the endplate, then the slot retains its parallel, resonant, equivalent circuit and the slot impedance is now the terminating impedance for the waveguide. Thus the slot presents a parallel impedance to the dominant mode in the guide. This is illustrated in Fig. 2(a)-2(d). In this analysis we are concerned with the impedance which the slot and waveguide system presents to the terminals of the tunnel diode at the center of the slot. In other words, it is important to deduce the relationship between the wave impedance of the slot as seen from the input end⁵ of the waveguide and the driving point impedance of

⁵ In the equivalent circuits presented here which refer to a waveguide input it is assumed that the input terminals are an integral number of half wavelengths from the endplate, or that the lumped equivalent circuit refers to the plane of the slot.

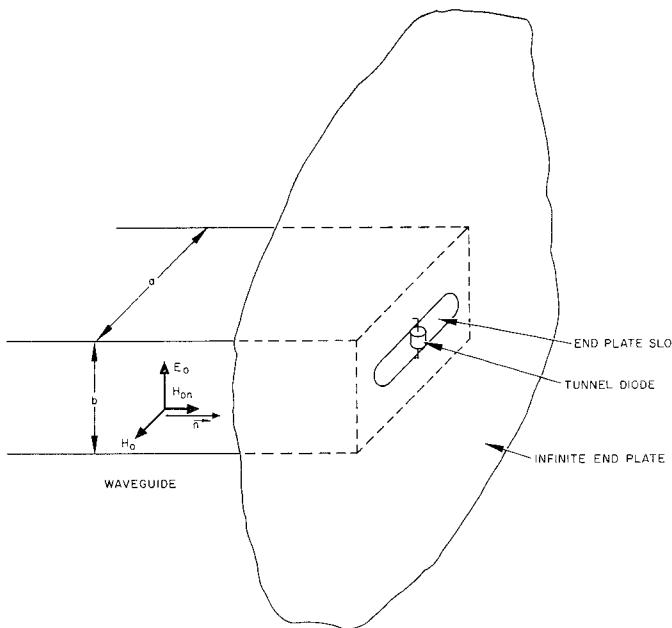


Fig. 1—Endplate slot containing a tunnel diode.

the slot as seen from a pair of terminals placed near the centers of the long walls of the slot [see Fig. 2(c) and (d)]. Thus the endplate slot acts simultaneously as a resonant circuit and as an impedance transformer. This impedance transformation ratio will be derived in a later section.

The microwave equivalent circuit of the tunnel diode has been described in the literature by several authors^{6,7} and is shown in Fig. 3 where the negative conductance is determined by the dc current-voltage characteristics of the diode shown in Fig. 4. By placing the tunnel diode across the slot (terminals ZZ'), one obtains the equivalent circuit shown in Fig. 5. If the waveguide is terminated by a constant current generator with an internal impedance equal to Z_g , the characteristic impedance of the waveguide for the dominant mode, the equivalent circuit of the system referred to the center of the slot is given by Fig. 6 for the frequency of resonance of the slot. At frequencies far removed from resonance this equivalent circuit is not valid. This, then, is the circuit describing the transmission amplifier, and it is easily converted to the receiving amplifier by transposing the constant current generator to the position of r_s . Thus, a wave incident on the slot from the right will induce a current in the system across terminals ZZ', and the magnitude of this current will be determined in part by absorption cross section of the slot. Fig. 7 demonstrates the lumped parameter equivalent circuit for the receiving amplifier and the feasibility of increasing the source and load currents by means of the negative con-

⁶ H. S. Sommers, Jr., "Tunnel diodes as high frequency devices," *PROC. IRE*, vol. 47, pp. 1201-1206; July, 1959.

⁷ B. Sklar, "Tunnel diode, its action and properties," *Electronics*, vol. 32, pp. 54-57; November, 1959.

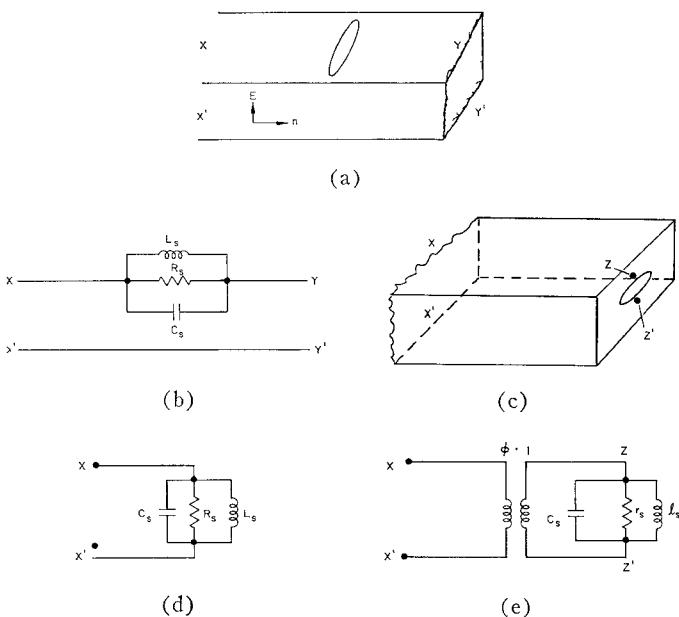


Fig. 2—Geometrical representation and circuit equivalents of slots. (a) Series slot in broad wall of a waveguide. (b) Equivalent circuit of series slot in broad wall. (c) Shunt slot in endplate of a waveguide. (d) Equivalent circuit of shunt slot in endplate of a waveguide. (e) Equivalent circuit of endplate slot showing impedance transformation property of slot at resonance.

ductance which is transformed by the reactive network to terminals ZZ'.

The preceding qualitative lumped parameter equivalent circuit analysis has been presented in order to provide the reader with some physical insight into the amplificatory behavior of this device and is based largely on heuristic arguments. The equivalent circuit for the slot is valid only near resonance, and the presence of an obstruction in the center of the slot such as a tunnel diode provides a sufficient lumped reactance to alter the coupling of the slot to the waveguide. This results in a shift in slot resonant frequency and an alteration of the elements of the equivalent network. If the dimensions of the diode become appreciable with respect to slot wavelength the fields will be distorted and the equivalent circuit parameters will be altered.

In view of the fact that the circuit is valid only over a narrow frequency range and must be changed for each higher order resonance of the device, it is of limited usefulness for determining stability conditions over a wide range of frequency. However, the lumped parameter equivalent circuit can be used to estimate the gain bandwidth product of the system.

Properties of the Resonant Endplate Slot Terminating a Waveguide: Stevenson⁸ has shown that for the case of a slot in one wall of an infinite (terminated) waveguide the voltage amplitude P at the center of the slot near resonance is related to the amplitude A of an incident TE_{10} wave by the formula

$$P/A = \xi/Ka, \quad (1)$$

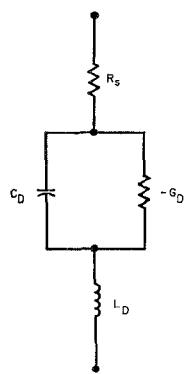


Fig. 3—Equivalent circuit of tunnel diode biased into the negative-conductance region.

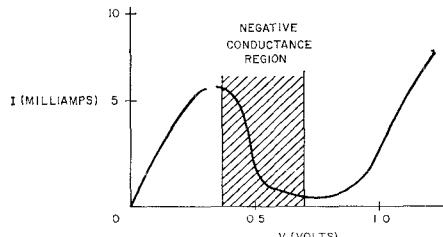


Fig. 4—Typical current vs voltage characteristic of a tunnel diode.

where ξ and K are complex dimensionless quantities defined in (2) and (3) and a is the broadwall dimension of the waveguide.

$$\xi = \int_{-\lambda/4}^{\lambda/4} f(\xi) \cos k\xi d\xi, \quad (2)$$

where $f(\xi)/a$ denotes the component of H along the slot at the point ξ in an H_{01} (TE₁₀) wave of unit amplitude traveling in the positive Z direction.

$\text{Re}(K)$

$$= -73/60\pi - (\gamma/2) |\xi|^2 \text{ (endplate slot only)} \quad (3a)$$

$\text{Re}(K)$

$$= -\gamma |\xi|^2 \text{ (guide to guide endplate coupling slot),} \quad (3b)$$

where $\gamma = 1/\pi^2 k U a b$, $k = 2\pi/\lambda$, $U = k_g = 2\pi/\lambda_g$, and b is the height of the guide.

The amplitudes of the reflected and transmitted waves in the guide generated by the voltage induced in the slot are given by

$$B/P = \xi/\pi^2 k U b$$

$$C/P = \xi^*/\pi^2 k U b \text{ (for guide to guide case only),} \quad (4)$$

where the star denotes the complex conjugate.

Applying these equations to the case of the waveguide terminated by an endplate slot one finds that the wave reflected from the endplate consists of two components. One component is due to the endplate in the absence of the slot and the other due to the presence of the

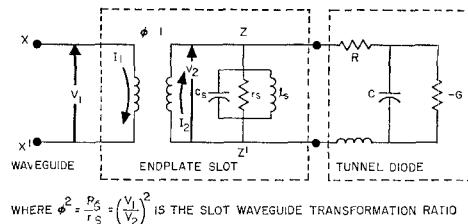


Fig. 5—Equivalent circuit of tunnel diode slot configuration referred to waveguide input terminals.

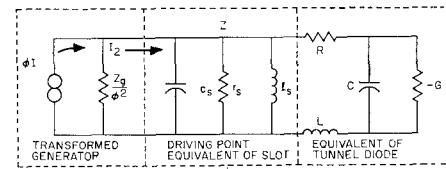


Fig. 6—Equivalent circuit of tunnel diode slot transmission amplifier including matched generator at waveguide input.

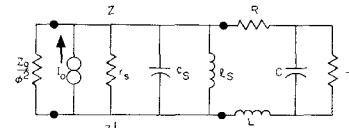


Fig. 7—Equivalent circuit of tunnel diode receiving amplifier.

slot. Thus, if the amplitude of the incident wave is A and the reflection coefficient of the plate is -1 , one obtains for the reflected component from (1) and (3)

$$B' = -A + B = -A \left(1 - \frac{\gamma\xi^2}{K}\right) \quad (5a)$$

or

$$\Gamma = \frac{B'}{A} = -1 + \frac{\gamma\xi^2}{K}, \quad (5b)$$

where Γ is the reflection coefficient of the system referred to the plane of the slot.

The impedance of the endplate slot normalized to the waveguide is then defined by

$$Z_n = \frac{1 + \Gamma}{1 - \Gamma} = \frac{\gamma\xi^2}{2K - \gamma\xi^2} = \frac{-30\pi\gamma\xi^2}{73}. \quad (6)$$

Consider now an endplate containing a slot of length $2L$ and width 2ϵ with its center displaced by x_0 from the narrow wall of the guide and oriented at an angle θ relative to the broadwall (see Fig. 8). Letting the X axis be parallel to the broadwall, one obtains from Stevenson's solution for the fields in the guide the magnetic component H_x

$$H_x = \left(j \frac{\pi}{a} U_{01}\right) \sin \frac{\pi x}{a}. \quad (7)$$

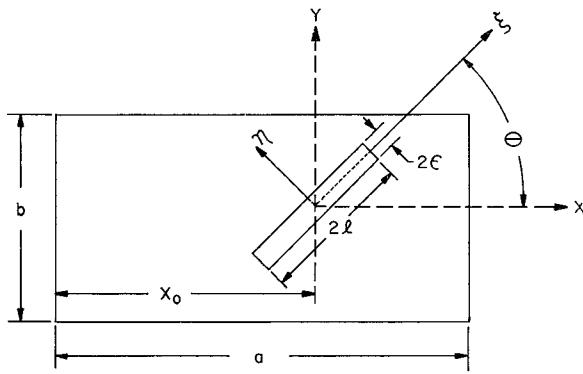


Fig. 8—Endplate slot position and orientation.

The component of H along the slot then has the form

$$\begin{aligned} \frac{f(\xi)}{a} &= 2H_{xi} = 2H_x \cos(\theta) \\ &= \left(j \frac{2\pi}{a} U_{01} \right) \sin \left(\frac{\pi(x_0 + \xi \cos \theta)}{a} \right) \cos \theta, \quad (8) \end{aligned}$$

where the factor 2 in front of H_{xi} is to account for the incident and reflected fields at the endplate. Combining (8) and (2), one obtains

$$\xi = -j4\pi \frac{U_{01}}{k} \cos \theta \left[\frac{\sin \left(\frac{\pi x_0}{a} \right) \cos \left(\frac{\pi}{4} \frac{\lambda}{a} \cos \theta \right)}{1 - \left(\frac{\lambda}{2a} \right)^2 \cos^2 \theta} \right]. \quad (9)$$

For the centered horizontal endplate slot the parameters X_0 and θ have the values $x_0 = a/2$, $\theta = 0$, and the function ξ is given by

$$\xi = -j4\pi \frac{U_{01}}{k} \frac{\cos \frac{\pi \lambda}{4a}}{1 - \left(\frac{\lambda}{2a} \right)^2} = -j4\pi \frac{\cos \left[\frac{\pi}{2} \sin i \right]}{\cos i}, \quad (10)$$

where $\sin i = \lambda/2a$ and $\cos i = \lambda/\lambda g = U_{01}/k$.

In general, the endplate function $|\xi|^2$ has the form

$$|\xi|^2 = \frac{16\pi^2 \cos^2 i \cos^2 \theta \sin^2 \frac{\pi x_0}{a} \cos^2 \left(\frac{\pi}{2} \sin i \cos \theta \right)}{[1 - \sin^2 i \cos^2 \theta]^2}. \quad (11)$$

The normalized impedance of the centered horizontal endplate slot thus becomes

$$\begin{aligned} Z &= \frac{30\pi}{73} \times \frac{1}{\pi^2 k U_{01}} \times \frac{16\pi^2 \cos^2 i \cos^2 \left(\frac{\pi}{2} \sin i \right)}{\cos^4 i} \\ &= 2.09 \frac{a}{b} \frac{\sin^2 i}{\cos^3 i} \cos^2 \left(\frac{\pi}{2} \sin i \right). \quad (12) \end{aligned}$$

The preceding analysis can be extended to the more general case of a displaced rotated endplate slot by the use of (9) and (6). (While this analysis is not new it ap-

pears in the literature in sufficiently ambiguous form as to require restatement here.)

Thus it is apparent that the slot presents an impedance to the reference plane of the waveguide given by

$$Z_{in} = Z_n Z_g = \left(\frac{\gamma \xi^2}{2K - \gamma \xi^2} \right) Z_g, \quad (13)$$

where Z_g is the characteristic impedance of the guide for the dominant mode of propagation, and Z_{in} is the waveguide input impedance.

Hence the slot behaves as an impedance transformer which transforms the driving point impedance of the resonant slot radiating into a half space ($2R_s$) into the input impedance given by (13) above. This impedance transformation ratio ϕ^2 is defined as the quotient of the input impedance Z_{in} and the slot impedance $2R_s$,

$$\phi^2 = \frac{Z_{in}}{2R_s} = \left(\frac{\gamma \xi^2}{2K - \gamma \xi^2} \right) \frac{Z_g}{2R_s}, \quad (14)$$

where

$$R_s = \frac{(120\pi)^2}{4 \times 73} \quad (15)$$

for a halfwave resonant slot.⁸

An equivalent circuit representation of this transformer is given in Figs. 8 and 9. If the waveguide is terminated in a matched load, the impedance presented to the driving point of the slot due to this load is given by

$$Z = \frac{Z_g}{\phi^2} = \left(\frac{2K - \gamma \xi^2}{\gamma \xi^2} \right) 2R_s. \quad (16)$$

Since this impedance is in parallel with the slot radiation resistance $2R_s$, the total driving point admittance of the resonant endplate slot coupled to a matched waveguide is found to be

$$Y_s = \frac{1}{2R_s} \left(1 + \frac{\gamma \xi^2}{2K - \gamma \xi^2} \right). \quad (17)$$

By applying conservation of power⁹ to the guide-to-guide mutual endplate slot, i.e., the resonant transverse iris, the driving point impedance of the slot radiating in one direction is found to be equal to

$$Z_{slot} = \frac{2(377)}{\gamma(\xi)^2} = 2\eta/\gamma(\xi)^2. \quad (18)$$

⁸ Putnam in the book noted below has shown that experimental values for the driving point impedance of resonant slots are centered about 350 ohms, and that the spacing of the centered driving points has a pronounced influence on this impedance.

⁹ J. L. Putnam, "Input impedances of centre-fed slot aerials near half wave resonance," *J. IEE (London)*, vol. 95, pt. 3A, p. 290; 1948.

⁹ By assuming a wave of unit amplitude incident on the slot, one can compute the power incident on the slot, the power reflected from the slot, and the power transmitted through the slot, as well as the voltage induced at the center of the slot. Then equating the transmitted power in terms of the slot voltage and impedance to the net power in the dominant mode we may obtain (18).

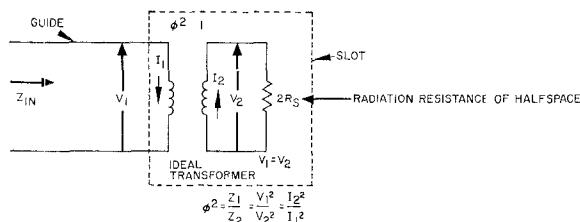


Fig. 9—Equivalent transformer—endplate slot.

If the resonant slot feeds a matched guide, all of the energy incident on the slot is transmitted and the input impedance is equal to Z_g . Therefore, a transformation ratio¹⁰ $(\phi')^2$ can be defined by

$$(\phi')^2 = \frac{Z_g}{2\eta/\gamma(\xi)^2} = \left(\frac{\gamma(\xi)^2}{2\eta} \right) Z_g. \quad (19)$$

(see Fig. 10). Thus the matched load Z_g transforms into the slot as $Z_g/(\phi')^2$. Adding this transformed impedance to that of the diode Z_D one gets

$$Z = \left(\frac{1}{Z_D} + \frac{(\phi')^2}{Z_g} \right)^{-1}. \quad (20)$$

Transforming back to the input yields the input impedance Z_{in} or admittance Y_{in} ;

$$Z_{in} = (\phi')^2 \left[\frac{1}{Z_D} + \frac{(\phi')^2}{Z_g} \right]^{-1}$$

$$Y_{in} = \frac{Y_D}{(\phi')^2} + Y_g = Y_g \left(\frac{2\eta Y_D}{\gamma(\xi)^2} + 1 \right). \quad (21)$$

Eq. (21) is an approximation relating the measured waveguide input admittance to the admittance of the diode in the waveguide slot. It is understood here that the transformation ratio $(\phi')^2:1$ relates the driving point impedance of the unobstructed guide-to-guide coupling slot to the input impedance of the waveguide. Any change in boundary conditions at the slot such as the presence of the diode or a change in guide height will alter this transformation ratio. However, if the circuit is analyzed from the viewpoint that the tunnel diode located at the center of the slot "sees" a constant current generator of internal impedance $Z_g/(\phi'')^2$ on the left and a load of impedance equal to $Z_g/(\phi'')^2$ on the right, where $(\phi'')^2$ is the true impedance transformation ratio, then the circuit shown in Fig. 11 will adequately represent the system, provided ϕ' is replaced by ϕ'' . Two problems preclude a meaningful analysis of the equivalent circuit of the slot tunnel diode guide-to-guide coupler. The first of these is based on the fact that at resonance it is difficult to compute the exact transformation ratio $(\phi'')^2$, and the second arises because it is difficult to determine the transformation ratio at fre-

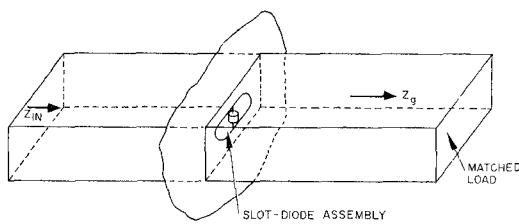


Fig. 10—Sketch of endplate slot—diode assembly acting as a guide-to-guide coupler.

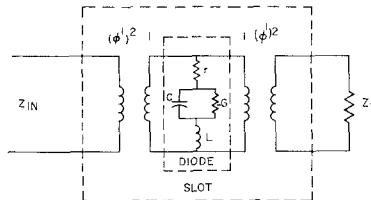


Fig. 11—Equivalent circuit of endplate slot diode guide-to-guide coupler.

quencies removed from the fundamental slot resonance.

As a consequence of the above analysis, the conclusion was drawn that ordinary waveguide measurements made on endplate slots and guide-to-guide coupling slots containing negative resistance diodes would yield information which could not be readily correlated with theory at frequencies off resonance. Moreover, at resonance the theory could be evaluated only by means of a parameter ϕ'' which in turn was not simply deducible from experiment because of the peculiarities of negative impedances. However, the form of the equivalent circuit obtained should be amenable to gain and noise analyses. In the next section a method of analysis is presented which qualitatively predicts the behavior of diode slot systems and is applicable to waveguide slot systems as well as slots in infinite conducting planes.

Admittance Analysis

Admittance Analysis of Tunnel Diode Slot Systems: An evaluation of (15) and (18), which describes the driving point impedance of an endplate slot and a guide-to-guide coupling slot, respectively, shows that the driving point radiation resistances of the slot radiating into space and into a matched waveguide differ by approximately 30 per cent for the case where $\lambda = 1.4a$ and $a = 2b$ (typical values for these waveguide parameters). Therefore, one can expect that the admittance properties of both slot geometries will show similar behavior as a function of frequency with the exception that the conductance of the guide-to-guide coupling slot will exceed that of the endplate slot by 30 per cent. Apparently here the role of the environment of the slot is to determine the degree of coupling of the slot field to the boundaries and has little effect on the resonant frequency of the slot. However, in the case of an endplate slot backed by a resonant cavity, Putman⁸ has shown that the presence of the cavity gives rise to a doubling of the resistive component of the impedance for all

¹⁰ Use of (6), (10) and (15) in (14) indicates that ϕ' in (19) is identical to (ϕ) in (14). This implies that the field configurations on either side of the slot are independent of each other.

cavity dimensions, and a marked dependence of the reactance at all frequencies on cavity dimensions. This is attributed to the natural reactance of the cavity in shunt with that of the slot.

In the light of this information, it was concluded that the driving point behavior of a resonant slot radiating endwise into two matched waveguides would be quite similar to that of a resonant slot cut in an infinite conducting plane and radiating into space. Hence an admittance analysis of the free space slot diode systems should predict qualitatively the behavior of the waveguide endplate diode slot and the guide-to-guide slot diode coupler systems as a function of frequency.

Calculation of the Driving Point Admittance of a Resonant Slot Cut in an Infinite Conducting Plane: By using an application of Babinet's principle¹¹ it can be shown that the radiation (driving point) impedance of a strip dipole and its slot complement are reciprocally related by the expression

$$Z_s = \frac{(Z_0)^2}{4Z_w}, \quad (22)$$

where $Z_0 = 120 \pi$, Z_s is the impedance of the slot, and Z_w is the impedance of a strip dipole of the same shape and size as the slot. This relationship is not restricted to resonance conditions; hence, if the impedance of the strip dipole is known as a function of frequency, then one can calculate the corresponding complementary slot impedance as a function of frequency. The slot admittance can be calculated from (22) by inserting the equation

$$Y_s = \frac{4}{(Z_0)^2} Z_w, \quad (23)$$

where Y_s is the driving-point admittance of the slot.

One point to be considered here is that the usual tabulations of impedance data presented by King¹² and others have been obtained from calculations based on wire antennas of circular cross section, whereas the complement of a slotted antenna is a flat strip. Hallen¹³ has shown that if the flat strip antenna has a width equal to W then it is equivalent in cross section to a wire of radius

$$a = \frac{W}{4}. \quad (24)$$

Since impedance data is usually tabulated for various values of the ratio of antenna length to diameter ($2h/2a$), this aspect parameter for the slot complement

¹¹ H. G. Booker, "Slot aerials and their relation to complementary wire aerials (Babinet's principle)," *J. IEE (London)*, vol. 93, Part 3A, p. 620; 1946.

¹² R. W. P. King, "The Theory of Linear Antennas," Harvard University Press, Cambridge, Mass., p. 173; 1956.

¹³ E. Hallen, "Theoretical investigations into the transmitting and receiving qualities of antennae," *Nova Acta Regiae Soc. Sci. Upsaliensis*, ser 4, vol. 11, pt. 1, p. 1; 1938.

can be written as

$$\frac{h}{a} = \frac{4h}{W}, \quad (25)$$

where $2h$ is the slot length and W is the slot width. The thickness-length parameter is sometimes described by the function

$$\Omega = 2 \ln \frac{2h}{a}. \quad (26)$$

The equivalent dimension parameter h/a or Ω for the slot is thus calculated from (24) or (25) and is used to select the appropriate tabulated wire dipole impedance data from King.¹² This impedance data is then converted by means of (23) into slot admittance data.

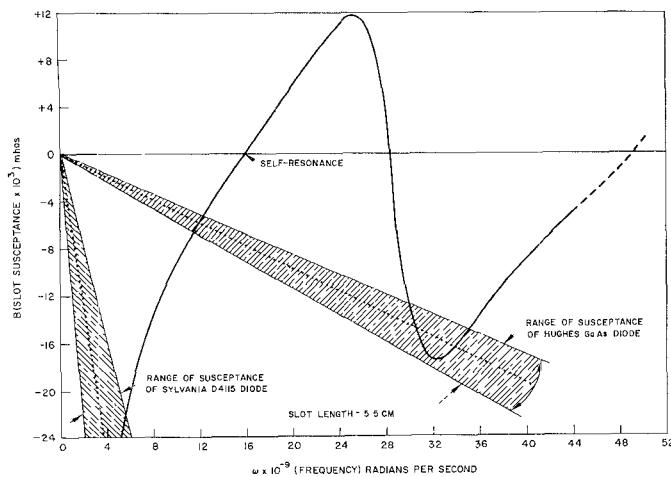
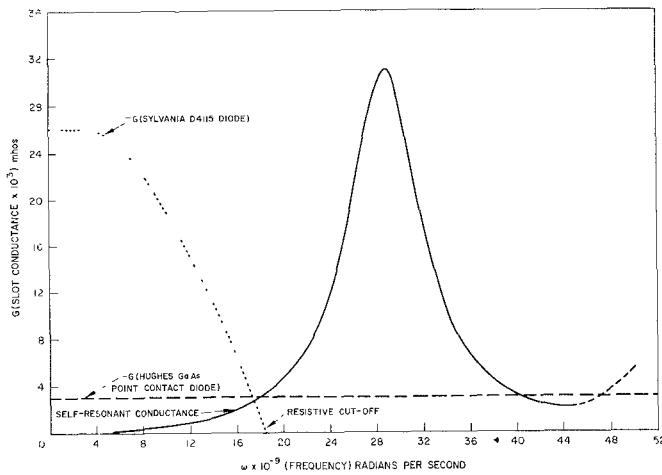
The device used in this experiment contained a slot of length 2.25 in and of width 0.030 in. Thus, from (25) and (26), the thickness-to-length parameters were found to be $h/a = 150$ and $\Omega = 2 \ln (2h/a) = 11.4$. Returning to the dipole data given by King, it was concluded that the resistance and reactance of a wire dipole of thickness-to-length ratio (h/a) equal to 122.4 was sufficiently accurate for use in evaluation of the slot admittance near the first resonance. Taking the real and imaginary parts of (23), the slot conductance and susceptance were computed from the relations

$$G_s = \frac{R_w}{35,476}$$

$$B_s = \frac{X_w}{35,476}, \quad (27)$$

where R_w and X_w are the real and imaginary parts of the dipole impedance, and G_s and B_s are the conductance and susceptance of the slot, respectively. The admittance data thus obtained is shown as a function of frequency in Figs. 12 and 13. Comparison of these graphs of the calculated slot conductance and slot susceptance as a function of frequency with measured values of slot resistance and reactance as a function of frequency obtained by Putman indicates that the theoretical and experimental curves have the same general form but differ slightly in scale factor. For example, the calculated conductance of the slot at resonance ($\lambda/2$) is 2×10^{-3} mho and is somewhat smaller than a value of 2.8×10^{-3} mho measured at lower frequencies by Putman. This discrepancy between theoretical and empirical values of resonant slot admittance has been discussed by Bailey¹⁴ who concludes that these values may differ by 20 per cent barring experimental inaccuracies and driving-point effects. The purpose of this comparison has been to show that the driving-point admittance as a function of frequency of a slot cut in an infinite conducting plane can be qualitatively predicted by

¹⁴ C. E. G. Bailey, "Slot feeders and slot aerials," *J. IEE (London)*, vol. 93, pt. 3A, p. 615; 1946.

Fig. 12—Slot susceptance as a function of ω .Fig. 13—Slot conductance as a function of ω .

means of the preceding computational steps. Moreover, the results of the calculation should be sufficiently accurate to enable prediction of the shifted resonance frequencies and the net driving-point conductances which will result when a microwave tunnel diode is placed across the center of the slot.

Conductance Transformation of a Tunnel Diode: If a semiconductor diode, such as a tunnel diode, is now placed at the feed terminals of the slot, the frequencies of resonance and the total conductance of the system at these resonance frequencies can be determined by superimposing the diode conductance and susceptance on the graphs of slot conductance and susceptance with the signs reversed. The resonance frequencies would, therefore, occur whenever the diode susceptance and the slot susceptance intersected.

In order to plot the diode susceptance and conductance in conformity with the foregoing scheme, one must first transform the equivalent circuit of the diode. By neglecting the series inductance of the tunnel diode equivalent circuit one obtains a three-element circuit which can be conveniently reduced to two elements (see Figs. 14 and 15). Since the impedances of both circuits

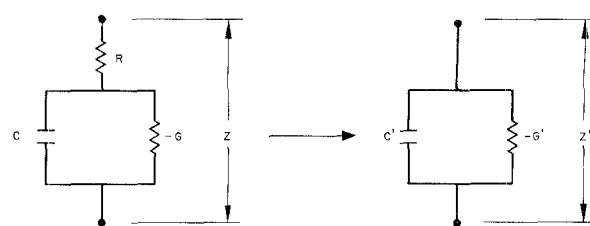


Fig. 14—Tunnel diode simplified equivalent circuits.

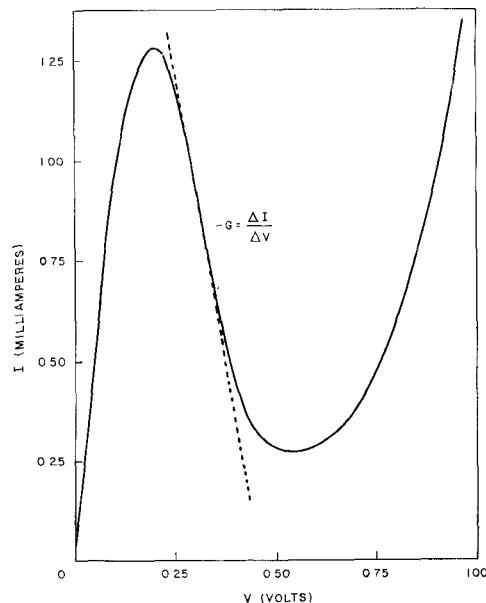


Fig. 15—I-V characteristic of Hughes PC-3 GaAs tunnel diode.

are equal, one can write

$$Z' = \frac{1}{j\omega C' - G'} = Z = R + \frac{1}{j\omega C - G}. \quad (28)$$

The resistive cutoff frequency $\omega = \omega(\max)$, occurs when $\text{Real } Z(\omega) = 0$. Solving for $\omega \max$ yields

$$f_{\max} = \frac{G}{2\pi C} \sqrt{\frac{1}{GR} - 1}, \quad (29)$$

where for later use we define

$$\alpha = f/f \max = \omega/\omega \max. \quad (30)$$

By equating the real and imaginary parts of (28) one obtains

$$C' = \frac{C}{(1 - RG)^2 + \omega^2 R^2 C^2} \quad (31)$$

$$-G' = \frac{-G + RG^2 + \omega^2 C^2 R}{(1 - RG)^2 + \omega^2 R^2 C^2}. \quad (32)$$

Using (29) and (30) to eliminate the capacitance C from the denominator of (31) for C' and from the numerator and denominator of (32) for $-G'$ yields

$$\frac{1}{G'} = \frac{1}{G(1 - \alpha^2)} - R, \quad (33)$$

and

$$C' = \frac{C}{(1 - RG)(1 - RG + \alpha^2 RG)} . \quad (34)$$

For frequencies far below cutoff $\alpha \ll 1$ and

$$\frac{1}{G'} \approx \frac{1}{G} - R \quad (35)$$

$$C' \approx \frac{C}{(1 - RG)^2} . \quad (36)$$

A typical calculation performed on the Sylvania D4115 B Tunnel Diode indicates that at low frequencies the negative resistance and the capacitance of the diode are given approximately by

$$\frac{1}{G'} \approx 36.4 \Omega \quad \text{or} \quad G' \approx 2.74 \times 10^{-2} \text{ mhos}$$

$$C' \approx 7 \text{ pf} \quad \text{for} \quad \omega < 19 \times 10^9 \approx \omega \text{ max.} \quad (37)$$

Plots of diode conductance G' and susceptance $-\omega C'$ are shown in Figs. 12 and 13 superimposed on the calculated values of slot conductance G and susceptance B . In addition, the estimated conductance and susceptance of the Hughes point contact Gallium Arsenide (PC-23) tunnel diode are shown on the same figures for comparison.

It is apparent from a study of Figs. 12 and 13 that at resonance the Sylvania tunnel diode cannot provide stable amplification because the negative conductance of the diode greatly exceeds the slot conductance, and will, therefore, give rise to large amplitude oscillations. Similarly in the case of the Hughes diode (PC-23) at resonance the negative conductance slightly exceeds the positive conductance and gives rise to oscillations.

Admittance Transformation Within the Slot: In order to stabilize the diode-slot system, some technique must be used to effect a transformation of the slot conductance and/or susceptance presented to the diode so that either the resulting conductance at resonance is positive or the resonant frequencies of the system are shifted into regions where the net conductances are non-negative. Several techniques are available which will cause the admittance presented to the diode to change. The first of these is the impedance or admittance transformation obtained when a dipole is fed asymmetrically. It can be shown that if the voltage distribution of an infinitesimally thin resonant slot antenna is independent of feed position, then the driving point resistance at resonance as a function of feed position is given by

$$R(d) = R_s \left[\sin \frac{\pi}{2} \frac{d}{l} \right]^2, \quad (38)$$

where $R_s \approx 480 \Omega$ is the radiation resistance of the half-wavelength slot when center fed at resonance, d is the position of the feed from one end of the slot, and $2l$ is the slot length. Inversion of (38) yields the resonant

conductance of the slot as a function of feed position

$$G_s(d) = \frac{1}{R_s \left[\sin \frac{\pi}{2} \frac{d}{l} \right]^2} . \quad (39)$$

It is apparent that the conductance presented to the diode at slot resonance can vary from approximately 2×10^{-3} mho with the diode at the center of the slot to ∞ mho with the diode at the end of the slot (for a thick slot the conductance at the end of the slot is finite).

According to King¹⁵ the input impedance of an asymmetrically-fed antenna is equal to the sum of the impedances of two antennas of different and quite arbitrary lengths. This conclusion is based on the evidence that the current distribution in each part of the antenna is largely independent of the other part. The antenna impedances under consideration here are half impedances calculated on the basis of center fed antennas of half lengths equal to the lengths of the asymmetrical elements. These half impedances are directly obtained from the impedance data computed for center fed elements by merely dividing the resistance and reactance by two. In fact the same impedance plot suffices to describe the impedance of these elements of dissimilar length provided the frequency scale on the abscissa of the data is adjusted to account for the different lengths. Thus, for an antenna fed near one end, one adds half of the impedance of a vanishingly small antenna to half the impedance of an antenna of almost twice the length of the total antenna. Further analysis of this system indicates that below resonance the effect of shifting the feed to one end is to increase the driving point impedance of the wire antenna. In a similar manner one finds that the admittance of a slot fed near one end is a large quantity near resonance for the slot. Moreover, this method of analysis of asymmetrical feeding of slots indicates that the net susceptance of the slot does not remain constant as one moves the driving point of the slot.

The conclusion drawn from the analysis of impedance transformation due to a shift in the driving point of the endplate slot is that for a given driving point (diode) location it is possible, using King's technique, to compute the driving point conductance and susceptance presented to the diode. However, if one requires the complete admittance behavior as a function of frequency for all slot positions, the data manipulation would require computer processing and plotting. In addition, the results would not possess a high degree of accuracy for determining system resonance frequencies because of the absence of susceptance data describing the local field perturbation due to the diode package within the slot.

A second technique for altering the slot admittance involves the use of a lumped admittance element placed within the slot. For example, by placing a rod through a hole cut in one wall of the slot, one can produce an obstacle within the slot whose behavior will be analogo-

¹⁵ King, *op. cit.*, p. 406.

gous to that of an obstacle in a waveguide. It has been experimentally determined that a slot which is resonant at 4 kMc can be suitably loaded by an obstacle so that it resonates at a frequency below 3 kMc. By applying Babinet's principle to cylindrical antennas it is observed that lengthening a half-wave dipole makes it more inductive, whereas lengthening a half-wavelength slot makes it more capacitive and shortening it makes it more inductive. Therefore, a slot cut to resonate at a given frequency will be inductive below resonance. Introducing an obstacle which incompletely fills the slot and provides the required amount of capacitance will restore the condition of resonance. It is thus apparent that by reducing the length of the slot one can shift the slot resonances to higher frequencies; then by placing a variable shunt capacitance at the center of the slot one can tune the slot in order to bring the fundamental resonance of the slot back to its original frequency. As a result of this manipulation the fundamental resonance will be unchanged, but all higher frequency resonances will be shifted to new frequencies, some higher and some lower than before. Furthermore, the resonant conductance of the slot will be reduced by this adjustment.

Returning to Figs. 12 and 13, it is seen that the presence of a lumped capacitance due to the diode or another obstacle at the center of the slot will shift the first resonance of the system to a lower frequency than one would obtain in the absence of capacitive loading. Therefore, if amplification is required at a given frequency, the slot must be designed so that the first resonance, after loading, falls at this frequency and all other resonances occur at frequencies where the net conductances of the system are positive. In order to avoid a detailed cut and try procedure, it is possible to provide the slot with a variable capacitance at the center to facilitate tuning to resonance. Another useful variable parameter is the tunnel diode bias voltage. By carefully controlling this voltage, the diode conductance can be varied from its maximum negative value through zero to a positive value. Thus by manipulating a variable capacitance and the diode bias voltage, it is possible to tune an appropriately selected diode slot system into the condition of oscillation or of stable amplification.

III. EXPERIMENTAL PROGRAM

Slot Amplifier

The theoretical analysis presented earlier in this paper yielded insight into the behavior of tunnel diode slot amplifiers located in infinite conducting plates, but no concise theoretical development was obtained which would predict the behavior of the waveguide slot-diode amplifier system over a wide range of frequency. On the basis of experimental results presented by Watson¹⁶ it was observed that the normalized admittance of slots cut in waveguide was quite similar to that of slots in an infinite conducting plane with the exception of a scale factor over a wide range of frequency. Hence it was concluded that the stability analysis of the "free space"

slot-diode system also applied approximately to the waveguide endplate slot-diode and guide-to-guide slot-diode coupler systems. Therefore, the experiment which was chosen to test the theoretical hypotheses expounded earlier included the guide-to-guide endplate mutual coupling slot. This resulted in the simplest experimental configuration in which the slot would still manifest many of the properties of an antenna and at the same time eliminated the need for an anechoic room.

The experimental apparatus consisted of an *S*-band signal generator coupled into a slotted line through a directional coupler and a 20-db waveguide attenuator; and an *S*-band spectrum analyzer connected by means of a 40-db strip-line isolator and a variable waveguide attenuator to the opposite end of the slotted line (see Fig. 16). In practice, the endplate slot containing the tunnel diode was inserted between the slotted line and the variable attenuator and the apparatus was adjusted to produce stable amplification. The slotted plate was then removed from the apparatus and the attenuator setting was changed to bring the transmitted signal back up to its previous value. The difference in these attenuator settings thus yielded the insertion gain of the system. In essence this method of measurement compares the transmissivity of the slot amplifier with that of a passive resonant slot, since the resonant slot will present a normalized conductance of unity to the incident wave in the guide.

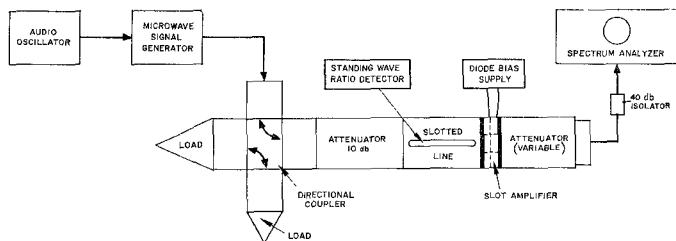


Fig. 16—Tunnel diode slot amplifier setup.

The endplate apparatus shown in Fig. 17 was originally designed to resonate unloaded at 2.7 kMc. However, the loading of the diode depressed the resonant frequency below the range of measurement of the apparatus. In the experiment performed in the laboratory, a second slotted endplate (designed to resonate above 4 kMc unloaded) with a movable obstacle in the slot was placed adjacent to and in contact with the semi-bifurcated endplate containing the tunnel diode. By varying the position of the obstacle as well as the diode bias, a condition of resonance was encountered at which the slot exhibited transmission gains of 8, 16 and 21 db in the neighborhood of 2.7 kMc with different diodes, and a 3 db bandwidth of 21 Mc was observed for the amplifier with a center frequency gain of 16 db. The frequency response plot is shown in Fig. 18. It was assumed at that time that the inductance furnished by the second endplate was sufficient to shift the first resonance into a frequency region where the slot conductance exceeded the diode negative conductance. However, the auxiliary slot shortened the original slot containing the

¹⁶ Watson, *op. cit.*, pp. 82-85.

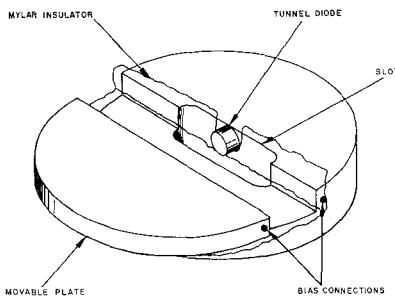


Fig. 17—Solid-state antenna configuration.

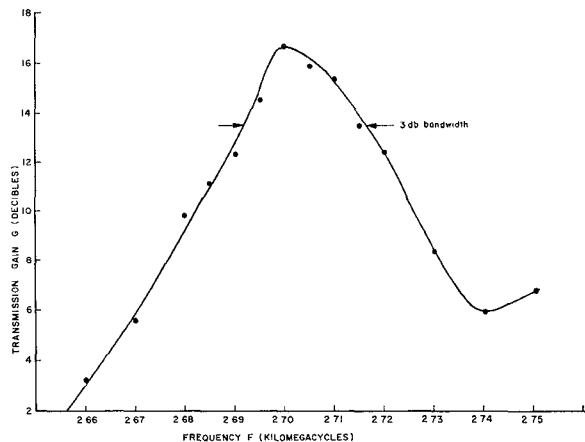


Fig. 18—Transmission gain as a function of frequency for the tunnel diode slot amplifier.

diode thus elevating its resonant frequency and influencing the positions of all system resonances in some manner as well as the conductance of the system as a function of frequency. It is obvious from the amplifying behavior of the system that this technique does not shift any higher resonances into negatively conducting regions; otherwise the power gain of the system at the first resonance would be negligible.

Slot Oscillator

During the course of the experimentation on active slot elements, it was observed that when a diode (Sylvania D4115 B) was located off center and suitably biased, the slot system would radiate a -38 dbm microwave signal in the neighborhood of 3 kMc. This frequency was dependent on slot position as well as bias voltage, but the amplitude was essentially independent of these conditions over a wide range of frequency. When a low level signal (-60 dbm) from the microwave signal generator was fed into the system it was observed that if the two signals were a fraction of a megacycle apart the oscillator would lock onto the incoming signal. This behavior was amplitude dependent, since very weak signals would not "pull" the oscillator, and very strong ones would give rise to harmonic generation and mixing.

IV. CONCLUSIONS

It is beyond the scope of this paper to analyze in detail the complete frequency dependence of system admittance because of a lack of adequate data describing the electrical parameters of the experimental tunnel diodes used in the experiment and because of the limited accuracy of the values of slot admittances which are based on theoretical wire dipole impedance calculations. However, the analysis and the results presented thus far indicate that if the conductance and susceptance of a tunnel diode are known over a wide frequency range it is possible to design a slot environment which will insure stable system behavior. Also, the variability of the diode admittance as a function of bias voltage provides a method for adjustment of the system. The equivalent circuits are presented in a form suitable for gain bandwidth and noise analyses, but no attempt has been made here to optimize the performance of the device.

The experimental work conducted on the project to date has been conducted entirely within a waveguide system. On the basis of the observed behavior it is expected that similar results will be obtained when the active slot elements are tested as antenna elements which couple to "space." It is anticipated that in the amplifier case a multiple element active array will require isolation devices between elements to prevent feedback oscillation from occurring.

The results presented indicate that the "solid-state antenna" concept may play a significant role in future radar systems. The diode slot amplifier can be used in an amplifying array in which the phase shift of each element is controlled by the diode bias voltage or by means of a varactor diode shunting the tunnel diode. On the other hand, the behavior of the slot-oscillator indicates the possibility of using an array of tunnel diode slot oscillators backed by a common cavity as a short range solid state transmitting antenna. These slot devices are compact and require a minimum of waveguide circuitry and in some instances can be used to convert directly from a microwave frequency to a low radio frequency by using the tunnel diode as an auto-dyne down converter. By backing the slot amplifier with a suitable resonant cavity, it may be used in conjunction with a circulator to form a reflection amplifier, and by cascading slot amplifiers and isolators it should be possible to produce a slotted traveling-wave amplifier.

ACKNOWLEDGMENT

The author wishes to acknowledge Dr. W. H. Kummer for his encouragement and interest in this work, and Dr. A. T. Villeneuve for many rewarding hours spent in discussion of the properties of slotted antennas. To Dr. L. L. Bailin and many colleagues at Hughes Aircraft Company the author wishes to express his gratitude for their unstinting support.